

CLAN CONCEPT IN MULTIPARTICLE DYNAMICS AND THE NB "ENIGMA"

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A summary of main results on NB regularity and related clan concept since their first appearance in multiparticle dynamics is presented.

1 Introduction

The main scope of this short contribution is to call the attention on the impact of Negative Binomial (NB) regularity in our field as documented by the collection of Proceedings of International Symposia on Multiparticle Dynamics since 1972. It is also quite instructive to follow the evolution of the interpretation of NB regularity from its first appearance in cosmic ray physics in the sixties and in the accelerators region in the seventies, up to its discovery in the eighties in limited regions of phase space in the CERN $p\bar{p}$ energy domain, and then in the nineties in e^+e^- annihilation at LEP in samples of events of different topologies up to the very recent suggestion presented at this Symposium to associate NB Multiplicity Distribution (MD) to quark jets of different flavour. This perspective is motivated by two facts. First, NB regularity in e^+e^- annihilation seems to be better satisfied the more fundamental and elementary is the chosen level of investigation of multiparticle final states: NB regularity is less approximate for the MD of the most primitive object occurring in Multiparticle Dynamics, i.e., the single jet of fixed flavour, and becomes more and more approximate in going from single jet to sample of two jet events and finally to the full sample of events. Second, it should be noticed that observed approximations of the regularity at a certain level of investigation can be in part removed by properly weighting NB MD's occurring with an higher degree of accuracy in the elementary substructures present at a more primitive level. It is clear that if these facts would be confirmed also in other classes of collisions it would be quite natural to consider the single jet structure as an effective building block in multiparticle production as it is now in e^+e^- annihilation, and the study of more complex final states structures as the result of compositions of single jets whose evolution is controlled by NB MD's and related clan structure properties. These remarks give a deep and striking physical meaning to the 1979 discovery that QCD jets can be described in LLA in terms of Markov branching processes of NB type dominated by gluon

self interaction, the non-linear QCD vertex, and to the subsequent observation that clans at parton level are bremsstrahlung gluon jets. Accordingly, the statement that QCD parton shower formation is the main dynamical mechanism responsible for multiparticle production turns out to be a consequence of the intimate fractal structure of jets proposed in our field - once again - since 1979. Along this line of thought an integrated description of multiparticle correlations and MD's is possible. In addition generalized hadron parton duality as formulated in the NB framework could be indeed a highly simplified approximated hadronization prescription and opens a new interesting research line on hadronization models.

2 Facts

2.1 Prehistory (1966)

NB regularity is discovered in cosmic ray physics¹. NB MD is called Polya-Eggenberger distribution by the name of the scientists who applied it to the diffusion of contagious diseases in 1926. Fluctuations of the pionization component in hadron showers originated by a primary hadron at different primary energies E_0 are well described by NB MD's.

Violations of multiperipheral model predictions are clearly visible starting from $E_0 \simeq 30$ GeV. Interestingly the Authors suggest the analytical structure of standard NB parameters in terms of the primary energy E_0 , i.e., the average charged particle multiplicity \bar{n} and the parameter k , which is linked to the dispersion D by $D^2 = \bar{n} + \bar{n}^2/k$, turn out to be respectively

$$\bar{n}(E_0) \sim 1.8(E_0)^{1/4} \quad (1)$$

and

$$k(E_0) \sim 0.4(1 - e^{-1.8 \cdot 10^{-4} E_0}) \quad (2)$$

It follows that at $E_0 = 30$ GeV, $\bar{n} \sim 4$ and $k \sim 465$, whereas at $E_0 = 10^4$ GeV $\bar{n} \sim 18$ and $k \sim 3$, i.e., k is a decreasing function of the primary energy as \bar{n} increases. A large part of the story of NB regularity in *full phase space* is in a certain sense already contained in the just mentioned results. Very large k values suggest that the MD can be safely approximated by multiperipheral model predictions (a Poisson MD) and the onset of correlations among produced particles is controlled by the increasing of $1/k$ at very large primary energies. See also the proceedings of the Wuhan 1991 symposium for an extensive discussion on the subject.

2.2 NB regularity in the accelerator region (Zakopane 1972, Pavia 1973)

NB (Polya-Eggenberger) MD is suggested for describing n-particle MD in full phase space in a simple model of multiparticle production which was motivated by the attempt to describe observed deviations from multiperipheral model predictions in the accelerator region². NB MD is discussed at the Zakopane Symposium in June 1972. Among other interesting features it is pointed out that the new distribution obeys in a certain limit KNO scaling and that related n-particle correlation functions depend on two-particle correlation functions only. The explicit formula and its derivation are also given. Subsequent systematic analysis of final charged MD's in all available experiments in the accelerator region confirmed that NB MD is a very good candidate to describe MD in full phase space³. The main results of this accurate analysis have been presented at the Pavia Symposium in 1973. The analysis has been completed in 1977⁴. 53 experiments were examined in total. The general trend of NB parameters is the same observed in cosmic ray physics, i.e., \bar{n} and $1/k$ are growing as p_{lab} increases in all reactions involving protons on hadronic targets. In addition, at lower p_{lab} $1/k$ becomes negative after crossing the zero at approximately 30 GeV/c.

2.3 NB regularity in symmetric (pseudo)-rapidity intervals in $p\bar{p}$ energy domain and then in all classes of reactions (Kyriat 1985, La Thuile 1989)

NB regularity is rediscovered by UA5 collaboration and extended to symmetric pseudorapidity intervals in $p\bar{p}$ collisions at c.m. energy $\sqrt{s} = 200$ GeV, 546 GeV and 900 GeV. First results are presented by G. Ekspong at the Kyriat Symposium in 1985. After the impressive work by NA22, EMC, HRS and NA5 collaborations - so well summarized by N. Schmitz in his talk at the La Thuile Symposium in 1989 - the new statement is: MD's in all classes of collisions in full phase space and in symmetric rapidity intervals are quite well fitted by NB MD. One problem shows up in $p\bar{p}$ at 900 GeV. NB does not fit data so well in the largest rapidity intervals. A shoulder is visible in the experimental distribution.

2.4 Clan concept (Seewinkel 1987)

Clan concept has been introduced in order to interpret the wide occurrence of approximate NB regularity in MP production. A clan is a group of particles of common ancestor (Sippe in German language). Each clan contains at least one particle (by definition). Clans are independently produced (by assumption). This new concept is discussed at the 1986 Seewinkel Symposium⁵ and exten-

sively applied to experimental data in a series of papers by the same authors⁶. The distribution of particles in an average clan is logarithmic (or a weighted geometric) and its composition with independently emitted (i.e. Poissonian) clans leads to the observed NB MD.

$$G_{NB}(\bar{n}, k; z) = \left[1 - \frac{\bar{n}(z-1)}{k} \right]^{-k} \quad (3)$$

$$G_{NB}(\bar{n}, k; z) = G_{Poisson}[\bar{N}; G_{log}(\bar{n}, k; z)] = \exp [\bar{N} (G_{log} - 1)] \quad (4)$$

with $\bar{N} = k \log(1 + \bar{n}/k)$ and $\bar{n}_c = \bar{n}/\bar{N}$, the average number of clan and the average number of particles per clan respectively, i.e. $G_{NB}(\bar{n}, k; z)$ is a Compound Poisson Distribution (CPD) or discrete infinitely divisible distribution. At this level the introduction of the clan concept is essentially a change in the variables one suggests to observe in the multiplicity distribution, i.e.

$$\text{from } \binom{\bar{n}}{k} \quad \text{to} \quad \binom{\bar{N}}{\bar{n}_c} \quad (5)$$

Notice that $1/k$ is interpreted here as a measure of aggregation of particles into clans

$$\frac{1}{k} = \frac{P_1(2)}{P_2(2)} \quad (6)$$

i.e., it corresponds to the ratio of the probability to have two particles in the same clan, $P_1(2)$, over the probability to have the two particles into two separate clans, $P_2(2)$.

Finally it should be recalled that in more general terms $1/k$ is linked to the two particle rapidity correlations $C_2(y_1, y_2)$ and to the second order factorial cumulant, κ_2 , by the equation

$$\frac{1}{k} = \kappa_2 = \int C_2(y_1, y_2) dy_1 dy_2 \quad (7)$$

The deep connection among clan structure parameters and correlations is starting from these very simple remarks.

The analysis of MD's in terms of the new parameters (clan structure analysis) reveals new striking regularities:

- a. linear increase of \bar{N} with the rapidity cut, y_c , at fixed c.m. energy; as y_c approaches full phase space, \bar{N} is bending towards a constant value;
- b. energy independence of \bar{N} in a fixed rapidity interval in the region of linearity;
- c. \bar{N} is larger in e^+e^- than in hh and in lh reactions whereas \bar{n}_c is smaller in e^+e^- annihilation and lh than in hh reactions.

2.5 NB regularity in Monte Carlo calculations (La Thuile Symposium 1989 and then Perugia and Ringberg Workshops)

In order to have informations on MD's at final parton level at different c.m. energies in full phase space and in rapidity intervals, i.e., in a region where QCD predictions are poor, clan structure analysis has been applied to $q\bar{q}$ and gg systems as resulting in JETSET Monte Carlo calculations. Results of this search have been presented at the La Thuile Symposium in 1989 and in the same period at the Perugia and Ringberg Workshops. This study shows:

- a. final partons and charged hadron multiplicities in full phase space and in symmetric rapidity intervals are well fitted by NB MD's.
- b. Hadronic and partonic levels are not independent but linked by the following equations

$$\left. \frac{1}{k} \right|_{hadron} = \left. \frac{1}{k} \right|_{parton}, \quad \left. \bar{n} \right|_{hadron} = \rho \left. \bar{n} \right|_{parton} \quad (8)$$

with ρ a constant $\sim 2Q_0/1$ GeV, Q_0 is the parton virtuality cut-off, or, in terms of n -particle (parton) inclusive multiplicity distributions

$$Q_n \Big|_{hadron} (y_1, \dots, y_n) = \rho^n Q_n \Big|_{parton} (y_1, \dots, y_n) \quad (9)$$

which can be considered a generalization of preconfinement and defines the Generalized Local Parton Hadron Duality (GLHPD) hadronization prescription.

- c. The general trend of the average number of clans is at both levels consistent with what was previously discovered by clan structure analysis at hadron level. In addition the regularity is more impressive at parton level than at hadron level, i.e., where Altarelli-Parisi or Konishi-Ukawa-Veneziano QCD evolution equations predictions are not affected by the Monte Carlo hadronization prescription.

2.6 The shoulder structure puzzle and NB regularity in jets of fixed topology (La Thuile 1989, Moriond 1991, Wuhan 1991, Vietri 1994, Stara Lesna 1995)

As already mentioned in $p\bar{p}$ reactions UA5 collaboration noticed that MD of the full sample of events at c.m. energy $\sqrt{s} = 900$ GeV in largest pseudorapidity intervals violates NB regularity and shows a characteristic shoulder structure. The effect was reported by Fuglesang at the La Thuile Symposium. Later the same effect was also seen at LEP at $\sqrt{s} = 91$ GeV.

The conclusion of those who were afraid of the regularity was that NB regularity was dead.

At the same time more conservative people asked to look better at the data: shoulder effect might very well be due to the overlap of jets of different topologies i.e. the regularity should be tested at a more elementary level. The results of this new analysis in e^+e^- at LEP energy are presented at the XXVI Rencontres de Moriond in 1991 by V. Uvarov. Although the analysis is dependent on the jet selection algorithm (JADE algorithm was used) three important facts occur:

- a. the shoulder in e^+e^- annihilation is the result of the superposition of jets of different topologies;
- b. MD's in jets of different topologies are nicely fitted by NB MD's, i.e., the regularity which is violated in the full sample of events is approximately restored at a more elementary level of investigation;
- c. LEP data less approximate on sample of two jet events are fully consistent with HRS Collaboration data at lower c.m. energy.

In particular clan structure analysis reveals that standard trends for \bar{N} and \bar{n}_c are less approximate on the sample of two jet events than in the full sample. In addition by applying GLHPD backwards one sees new impressive regularities at parton level: the average number of clan at parton level is 1 in $\Delta y = 1$, 2 in $\Delta y = 2$, ..., 5 in $\Delta y = 5$. This fact is verified both at LEP and HRS c.m. energies. The average number of clans is a linear function of rapidity variable. The average number of partons (particles) per clan follows the standard trend. It should be stressed that the same mechanism, i.e., the superposition of jets of different topologies (hard gluon radiation) explains in part also H_q oscillations.

2.7 New results (Faro 1996)

Two new results have been presented at this Symposium. a. H_q oscillations and NB regularity in quark jets of fixed flavour. In e^+e^- annihilation at the Z^0 peak either the charged particle MD, P_n , or H_q oscillations are well reproduced in full phase space by the weighted superposition of two NB MD's associated to $b\bar{b}$ and light flavoured events, the weight being given by the fraction of $b\bar{b}$ events. The corresponding NB parameters are characterized by the fact that⁷

$$\bar{n}_{light\ flavoured\ events} \neq \bar{n}_{b\bar{b}\ events} \quad , \quad k_l = k_b \quad (10)$$

Clan structure analysis suggests that the aggregation of particles into clan is the same in $b\bar{b}$ and light flavoured events!

b. clan structure analysis at H1 experiment at HERA. H1 Collaboration at HERA presented data on MD's in different pseudorapidity windows at various energy intervals and then fitted these data by NB MD⁸. Clan structure analysis performed on these data reveals that the average number of clans in a fixed rapidity interval is energy independent and that the average number of particle per clan follows the same general trend already observed in EMC data (the figure presented at this Symposium is not shown here due to the lack of space).

3 Theory

3.1 Jets as QCD Markov branching processes (Goa 1979, Bruges 1980, Wuhan 1991, Vietri 1994)

QCD jets have been described in LLA as Markov branching processes of NB type since 1979⁹. The approach is a purely exclusive one and consists in solving explicitly KUV equations with a fixed cut off regularization prescription, once the intrinsic Markoffian structure of the process has been recognized. It should be recalled that KUV equations are deduced in an inclusive approach to the production process and are the differential formulation of the integral AP QCD equations¹⁰. Results of this search have been extensively discussed in Goa (1979), Bruges (1980), Wuhan (1991) and Vietri (1994) Symposia. The main aspect of this study is that parton showers behave here as self-similar fractals; they develop according to a principle of *disorder* (fractal from Latin *frangere*, to break) followed by a principle of *order* and controlled by self-similarity, i.e., according to an "Italian firework" of jets of jets... In other words in a quark parton generated shower the ancestor quark "breaks" into bremsstrahlung gluon jets (the clans) which then decay into final parton via gluon self interaction, and partons belonging to each clan are distributed according to a weighted geometric distribution. What is interesting to point out is that clan structure analysis decouples the $g \rightarrow g + g$ vertex from the $q \rightarrow q + g$ vertex and that $1/k$ is here just their ratio.

3.2 Clan structure analysis in statistical physics (Aspen 1993, Vietri 1994)

This study concerns:

a. clans and compound Poisson distributions¹¹; b. clans and hierarchical correlation functions¹². It has been done in collaboration with Sergio Lupia and Roberto Ugoccioni and reported in Aspen 1993 and Vietri 1994 Symposia.

a. The description of multiparticle production in a domain of rapidity Δy as a two step process can be formulated in very general terms in the framework of infinitely divisible distributions (IDD). Any discrete IDD is indeed a

Compound Poisson Distribution (CPD). In the generating functions formalism this important property leads in a natural way to the generalized clan concept (g-clan) as can be seen by inspection of the following general definition of the generating function of any CPD:

$$G_{CPD}(z; \Delta y) = \exp [\bar{N}_g(\Delta y)[G'(z; \Delta y) - 1]] \quad (11)$$

where \bar{N}_g is the average number of g-clans and $G'(z; \Delta y)$ is the generating function of the MD inside an average g-clan. Since by definition each g-clan contains at least one particle the condition $G'(z; \Delta y)|_{z=0} = 0$ has to be imposed. Accordingly clan concept is more general than NB multiplicity distribution, whose wide occurrence in final particle multiplicity distributions justified its first introduction. In addition a simple relation exists for any CPD among the probability $P_0(\Delta y)$ to detect no particle in a given rapidity interval Δy and the average number of generalized clan, \bar{N}_g , i.e.

$$G_{CPD}(0; \Delta y) = e^{-\bar{N}_g(\Delta y)} = P_0(\Delta y) \quad (12)$$

b. The main result can be summarized by the following theorem: factorial cumulant structure is hierarchical, i.e., n -order factorial cumulants, $\kappa_n(\Delta y)$, are controlled by second order factorial cumulants, $\kappa_2(\Delta y)$, only *iff* the function $\mathcal{V}(\Delta y) \equiv -\log P_0(\Delta y)/\bar{n}(\Delta y)$ scales with energy and Δy as a function of $\bar{n}(\Delta y)\kappa_2(\Delta y)$. Notice that the just mentioned scaling function is the inverse of the average number of particles per generalized clan, $(\bar{n}_c)_g$.

In conclusion clan structure parameters can be extended to the class of discrete IDD's (they are the most economic way to describe a two step production process in which independently generated extended objects – clans, clusters, strings, ... – decay into final particles or partons). In addition these parameters are useful and intriguing in view of their deep physical meaning (they control "voids" distribution in rapidity and being $\kappa_n(\Delta y) = \int_{\Delta y} dy_1 \dots \int_{\Delta y} dy_n c_n(y_1, \dots, y_n)$ the hierarchical content of n -particle (-parton) correlation functions $c_n(y_1, \dots, y_n)$ in the production process). Therefore the scheme described in (5) should be completed as follows

$$\text{from } \left(\frac{\bar{N}(\Delta y)}{\bar{n}_c(\Delta y)} \right) \quad \text{to} \quad \left(\frac{P_0(\Delta y)}{\mathcal{V}(\Delta y)} \right) \quad (13)$$

In case of the NB MD the new set of parameters is linked to the previous one by the equations

$$\bar{N}(\Delta y) = -\log P_0(\Delta y) = -\log \left[\frac{k(\Delta y)}{\bar{n}(\Delta y) + k(\Delta y)} \right]^{k(\Delta y)} \quad (14)$$

$$\bar{n}_c(\Delta y) = \frac{\bar{n}(\Delta y)\kappa_2(\Delta y)}{\log[1 + \bar{n}(\Delta y)\kappa_2(\Delta y)]} = \mathcal{V}(\Delta y)^{-1} \quad (15)$$

The second equation shows that NB regularity predicts that n -particle correlations are hierarchical¹².

3.3 Clan structure parameters in theoretical models (Vietri 1994, Stara Lesna 1995)

The energy dependence of clan structure parameters can be calculated analytically either in f.p.s. or in rapidity intervals in the generalized simplified parton shower model (GSPS)¹³. The model is a generalization of the work initiated with Leon Van Hove¹⁴. It is a two parameter model and is based on *essentials of QCD* (gluon self interaction) in a convenient kinematical framework: energy-momentum conservation laws are violated locally but not globally. Clans are considered here as independent intermediate fluctuating gluon sources. Observed general trends of clan structure parameter in energy and rapidity are correctly predicted^{13,15}. These results suggest that the ancestors of the clans, i.e., the independently emitted bremsstrahlung gluons, should be indeed identified at parton level with the just mentioned fluctuating gluon sources.

3.4 Clans and the hadronization problem (Stara Lesna 1995, Faro 1996)

At this point we are ready to ask the following question: are clans observable physical objects? A possible hint comes from the thermodynamical hadronization model¹⁶. The model has been successfully applied to hadron production in e^+e^- annihilation. The hadronic spectrum is quite well reproduced in terms of three parameters (the temperature, the volume and strangeness chemical suppression). The production process is described as a two-step process in which primary hadrons emitted by the thermal source decay into final particles. It is interesting to point out that final charged tracks MD's turn out to be of NB type and the average number of clans calculated from fitted NB coincides with the average number of primary hadrons predicted by the thermodynamical model¹⁷. This result suggests that at hadron level in e^+e^- annihilation clans should be identified with primary hadrons.

4 Conclusions

The main results of experimental and theoretical work done along the years on approximate NB regularity and related clan structure, and reported in Multiparticle Dynamics Symposia, have been summarized. The hope is to

have given a perspective for future work and a contribution to the answer of the following questions.

Is NB regularity still an enigma – as Leon Van Hove pointed out in Shandong in 1987 – or a useful Arianna’s thread – as I prefer now – in order to drive us in the labyrinth of multiparticle dynamics?

Is the fact that in e^+e^- annihilation NB MD can be associated to jets of different flavour accidental, or does it reveals the intimate structure of multiparticle dynamics in the building block of the reaction? If it is so, what about other reactions?

Is clan concept a purely statistical one (like cluster expansion in statistical mechanics) or are clans observable physical objects?

Are clans at parton level bremsstrahlung gluon jets? Can one consider the ancestors of each clan (the bremsstrahlung gluons) independently emitted fluctuating gluon sources?

Are really clans at hadron level in e^+e^- annihilation primary hadrons? If it is so, what about other reactions?

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